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JOUKOWSKI WINGS

By W. Margoulis.

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Professor Joukowski died in Moscow at the age of 72 years. Death overtook him at a time when recent experiments had just confirmed his theories with regard to supporting wings and demonstrated the remarkable properties of the wing sections he had proposed. It is this confirmation of the theories of my teacher and their importance from the standpoint of aviation that I propose to set forth in the present article.

I. The Joukowski Wings.

A. Consideration of the recent experiments in the Göttingen laboratory on the aerofoils and especially the wing sections of Joukowski.

The Göttingen laboratory has just published** the results of its tests of numerous wings, made either by the ordinary method with models 200 × 1000 mm., with a velocity of 30 m/sec, or with models 600 × 1500 mm placed between two vertical walls, at velocities of 10 to 40 meters per second. The first series of tests were made with about a hundred different wing sections, the second with only six.

Fig. 1 gives the sections of the Joukowski wings, numbered 429-435, also No. 358 (likewise a Joukowski wing), and No. 390.

* From "L'Aéronautique," August, 1921.

** Prandtl, Wieselsberger and Betz, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," I Lieferung (First Report of Göttingen Aerodynamic Laboratory), pp. 54-71. Figs. 1-3 were taken from the above work.

Fig. 2 gives the polars of wings 429-435 obtained from the 300 × 1000 mm models at 3 m/sec. Aspect ratio is 5. Fig. 3 shows the results obtained with wings 358 and 390, both between parallel walls, with velocities of 10, 25 and 40 m/sec, and also by the ordinary method, at 10 and 30 m/sec. Aspect ratio is 5.

Prandtl's theory of the induced drag made it possible to determine the theoretical aspect ratio (which was 4.1) of the surfaces tested between two walls and to reduce the results to the aspect ratio 5. The polars, obtained by the two methods with the same Reynolds numbers, agree well, especially for wing 390, thus justifying the method of calculating the theoretical aspect ratio.

The results of the Göttingen tests on aerofoils, in consequence of which experiments have been made with Reynolds numbers higher than any heretofore realized, throw new light on a subject which until recently was somewhat unknown.

We will first recall the conclusions reached only a few months ago, when we examined the results of aerofoil tests made with a product Vl of the Reynolds number equal to 1 m^2/sec.*

1. A comparison of the polars of thin wings (maximum relative thickness less than 10%) showed that they were drawn on the assumption of a parabolic curve parallel to the parabola of the induced drag. The sectional drag indicated by this curve was practically constant and slightly greater than the drag due to friction alone. The polars of the thin flat wings followed the curve for small

* See our article "L'Aérodynamique experimentale en 1930: Les Voilures," in the January, 1921, number of L'Aéronautique.

lift coefficients and diverged from it for large lift coefficients, giving a rather small maximum C_a. The polars of thin cambered wings closely followed this curve for large lift coefficients, showing a high maximum value of C_a, but differing appreciably from the curve for small lift coefficients. Whatever the camber of the wing, its polar closely followed the ideal curve within relatively narrow limits of the lift coefficients.

2. An examination of the curve of the polars of wings of average thickness (10 to 15%) makes it possible to demonstrate that the sectional drag of this curve was greater than that of the curve for thin wings, but that (a very important fact) the polars of certain cambered wings of average thickness followed the ideal curve throughout very broad limits of the lift coefficients. An increase in thickness tended to eliminate the deflection for small lift coefficients.

3. Lastly, the sectional drag of the ideal curve of the polars of thick wings (over 15%) was appreciably greater than that of the curve for wings of average thickness. There was likewise demonstrated for the polars of these wings a tendency to follow the ideal curve closely, although in a less degree, chiefly on account of the deflection for large lift coefficients.

To sum up, thin wings practically realize (though within very narrow limits of the lift coefficients and varying for the different wings) the ideal flow without deflection of the air stream. Wings of average thickness do not realize this flow without deflection, but approximate it within very broad lift limits. Lastly, thick wings diverge considerably from the ideal flow.

From the practical point of view of the choice of the wing section, this state of affairs, that is, of the non-existence of one wing section superior to all others for all lift coefficients had led us to a method based on a rapid determination of the performances of airplanes with different wing surface areas, corresponding to the imposed conditions, maximum speed for a given ceiling for military airplanes or maximum flying speed for a given landing speed for traffic airplanes. For certain military airplanes, however, the conditions imposed led to the consideration of the minimum landing speed.

In the above-cited article, we had been led to recommend thin flat wings for military airplanes and wings of average thickness and camber, especially the Göttingen wing No. 165, for commercial airplanes.

We had long insisted, however, on the uncertainty of laboratory results obtained with such low values of the Reynolds number. In fact, recent experiments in the Göttingen laboratory, with values for vl of 6 to 24 m^2 : sec, gave results very different from the former ones.

We shall consider in detail the conclusions which may be drawn from these new experiments, and we shall add former experiments at the Göttingen laboratory on four thin wings between vertical walls, for $vl = 30\ m^2$: sec (See "Zeitschrift für Flugtechnik und Motorluftschiffahrt," May 31, 1919, "Ähnlichkeiten an Flügelprofilen," by H. Kumbruch.)

1. As regards thin wings, we find few changes. The sectional drag of the ideal curve of their polars is slightly diminished, but the deflection of the air stream for small lift coefficients for cambered wings and for large lift coefficients for flat wings always persists. However, it seems possible to avoid the deflection for small lift coefficients of a thin wing of average camber, by raising its trailing edge (See wing 393, which is the famous Lanier-Lawrence).

2. Wings of average thickness gain the most by increasing the value of the Reynolds number. For $vl = 6\ m^2 : sec$, the ideal curve of their polars coincides practically, and for $vl = 24\ m^2$ sec exactly, with the curve of the polars for thin wings.

Furthermore, the polars of some of these wings, especially those of the Joukowski wings (Nos. 430 and 358) coincide perfectly with this curve. The increase in the value of the Reynolds number eliminated the deflection of the air stream and produced a perfect flow for all lift coefficients. Fig. 3 clearly shows the disappearance of the deflection of the air stream for both small and large lift coefficients proportional to the increase of vl.*

3. For thick wings, we find a deflection for large lift coefficients leading to small maximum values of C_a. Moreover, this

* These results show once more the uselessness of "comparative" tests with small values of Reynolds number, since, under these conditions, a thin wing and one of average thickness can give equal drags, while offering very different drags for high values of the Reynolds number.

maximum value of C_a, instead of increasing when vl increases, as happens for wing 358, diminishes, as shown for wing 390.

We can readily perceive the importance of these results. For the first time in experimental aerodynamics, we find ourselves in the presence of a series* of wings superior to others for all lift coefficients.

From the practical point of view in choosing a wing section for an airplane, they lead to the adoption in all cases, of wings of average thickness and camber, and especially of Joukowski wings.

B. Comparison of the theoretical values of the lift coefficient deduced from the theory of Professor Joukowski with the experimental values.

It is known that the theory of the Joukowski wings renders it possible to determine the values of the lift coefficient in terms of the angle of attack for certain groups of wing sections whose characteristics he determined.

The Joukowski wings, tested in the Göttingen laboratory, all belong to the same family, with chord l (Fig. 4), obtained by representing in the plan of the variable complex $\zeta = \xi + i\eta$ by means of the transformation $\zeta = z + \frac{l^2}{4z}$, the circumferences situated in the plane $z = x + iy$ and passing all through the point $x = -\frac{l}{2}$. One will find in the French translation of Professor

* For $Vl = 6$, wing 430 gives a polar only slightly superior to that of wing 358. It is certain that for $Vl = 24$, these polars will continue to be practically the same, whence it follows that all the intermediate wing sections will give the same polar.

Joukowski's work: "Bases théoriques de l'Aéronautique, L'Aérodynamique," (Paris, Gauthier-Villars, 1916, Chap. VI), the complete exposition of this theory.

Professor Joukowski applied the above transformation to the circumferences whose centers are on the x-axis alone. He thus obtained symmetrical double convex sections like wing No. 429, for example.

Professor Blumenthal, after studying the pressure distribution on Joukowski wings, extended this transformation to every circumference whose central coordinates have positive values. He characterized these sections by the values $\frac{f}{l}$ and $\frac{\delta}{l}$, fixing the position of the center (Fig. 4).

The former of these values is equal to twice the maximum relative camber of the mean line of the wing. The latter characterizes the maximum relative thickness $\frac{e}{l}$ of the wing,

$$\frac{e}{l} = 2.2 \text{ to } 2.6 \left(\frac{\delta}{l}\right)$$

(For $\frac{\delta}{l} = 0$, the wing sections are arcs with cambers equal to $\frac{f}{2}$)

One will find in Fig. 5 these values for wings 358 and 429 to 435. As regards wing 358, let us note that it was not designed as a Joukowski wing. It seems to constitute a transformation (by rounding the tip Av) of wing 165, which already possessed the thin cambered rear position peculiar to the Joukowski wing sections. In fact, if a wing section is drawn from the data of Fig. 5, it will be practically like No. 358.

The value of the total lift P on a double convex wing of infinite aspect ratio, as given by Professor Joukowski, was

$$P = 2\pi\rho \, bs \, V^2 \sin\beta$$

in which ρ is the density of the fluid, b the radius of the transformed circumference, s the span, β the angle between the direction of the wind and the ξ axis.

In the general case, we found for the value of the lift,

$$P = 2\pi\rho \, bs \, V^2 \sin\left(\text{arc tang}\,\frac{f}{l} + \beta\right).$$

The equation for the lift coefficient is

$$c_a = \frac{P}{\frac{\rho}{2}\, ls \, V^2} = 2\pi \left\{\sqrt{1 + \left(\frac{f}{l}\right)^2} + \frac{2\delta}{l}\right\}$$

$$\times \sin\left(\text{arc tang}\,\frac{f}{l} + \beta\right).$$

For comparing the theoretical with the experimental lift coefficients, we proceeded as follows:

1. We traced, in Fig. 5, the curve MN giving the values of

$$c'_a = \frac{P}{\frac{\rho}{2}\, 2\, bs \, V^2} = c_a \frac{l}{2b} = 2\pi \sin\left(\text{arc tang}\,\frac{f}{l} + \beta\right)$$

in terms of the angles $\left(\text{arc tang}\,\frac{f}{l} + \beta\right)$ carried on the abscissas.

2. Let α_s and c_a be two correlative values of the angle of attack and of the lift coefficient given by the tests on a wing with an aspect ratio 5, the angle of attack being measured with reference to the straight line AA (Fig. 4) doubly tangent to the bottom camber and making an angle α_0 with the ξ axis. It is

known that the angle of attack α_∞ (expressed in degrees), giving the coefficient of lift c_a for a wing of infinite aspect ratio, is related to the angle α_s by the equation

$$\alpha_\infty = \alpha_s - \frac{c_a}{\pi \times 5} \times \frac{180}{\pi} = \alpha_0 - 3.64\ c_a.$$

Consequently

$$\text{arc tang}\ \frac{f}{l} + \beta = \text{arc tang}\ \frac{f}{l} + \alpha_0 + \alpha_s - 3.64\ c_a.$$

On the other hand, $c'_a = c_a \frac{l}{2b}$, in which c_a is the experimental value of the coefficient of lift for the angle α_s. We have indicated in Fig. 5 the values corresponding to the tests made with wings 358 and 429 to 435.

3. Fig. 6 gives the values of the ratios of the experimental to the theoretical coefficient of lift in terms of the sectional drag of the same wings.

These two figures lead us to the following conclusions:

a. The experimental coefficient of lift approaches the theoretical coefficient of lift in the same proportion as the total drag approaches the frictional drag or, in other words, as the flow takes place without deflection.*

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* According to the Göttingen tests, the coefficient of friction for surfaces of fabric with six coats of dope, would be

$$16\ K_f\ (kg/m^2/m : sec) = c_f = 0.0375\ N^{-0.15}$$

N being the Reynolds number, equal to the product of the velocity times the chord, divided by the kinetic coefficient of viscosity (which is $\frac{14.5}{10^6}$ m²: sec for air at 15°C and 760 mm Hg). We thus find for:

Vl	2	6	15	24	m² : sec.
c_f	0.00636	0.0054	0.0047	0.00437	
$200\ c_f = C_{wo}$	1.27	1.08	0.94	0.87	

(Contd. p. 10)

b. For the highly cambered wing with the angle

$$\text{arc tang}\,\frac{f}{l} + \beta = 2.5^\circ \quad (\alpha_s = -9.1^\circ),$$

the coefficient of lift, in spite of a very great sectional drag, approaches the theoretical coefficient of lift much more closely than with the angles of 5.3° to 8.2°. This indicates a change in the flow which may likewise be observed for many other wings tested at Göttingen and especially for the very thick wing 231, for which a negative value of the lift coefficient was found with the angle of attack $\alpha_s = -12^\circ$ and positive values of the lift coefficient with smaller angles of attack of -13° and -15°. This phenomenon is probably analogous to that produced with large positive angles of attack, as shown by a comparison of wings 382 and 383 or of 387 and 386.

c. For $Vl = 6\ m^2 : sec$, the mean ratio of the coefficients of lift is 0.77, insofar as there is no appreciable deflection, and excepting wing 429.

d. For $Vl = 24\ m^2 : sec$ (wing 358) and for wing 429 with small lift coefficients, this ratio is 0.91.

2. It is probable that, for values of Vl below 24, the actual flow still more closely approaches the theoretical flow.

We thus see the reason which prevented Professor Joukowski

* Contd. from p. 9)
If the sectional drag were due simply to friction and if the velocity were uniform over the whole surface, the coefficient of the sectional drag (C_{wo}) would equal twice the coefficient of friction. It is found, however, for certain very thin double convex wings, that the sectional drag is almost twice as small as the frictional drag. Thus, for wing 444, $C_{wo} = 0.56$, while $c_f = 1.08$, which results require verification.

and Mr. Betz (Untersuchungeiner Schukowskyschen Tragfläche, Zeitschrift für Flugtechnik und Motorluftschiffahrt, Sec. 24, 1915) from drawing definite conclusions from the theory of supporting wings. In fact, the tests made by the former in the laboratory of the imperial technical school at Moscow, and by the latter in the Göttingen laboratory, were based on too small values of Reynolds number and led to deflections with small coefficients of lift and very high sectional drags.* (In Professor Joukowski's tests, the angles of zero lift coefficient always differed widely from the theoretical values.)

To sum up, Professor Joukowski's theory of supporting wings renders it possible to calculate the coefficient of lift in terms of the angle of attack, and Prandtl's coefficient of induced drag and the correction of the angle of attack in terms of the disposition and aspect ratio of the wings. By adopting a sectional drag equal to the frictional drag, we can very closely approximate, by calculation, the elements of the resultant of an aerofoil with a given section.**

We are therefore approaching the solution of a problem which has troubled scientists since the beginning of aviation. la A large share of the credit for this solution will revert to Professor Joukowski. Thus the words of Helmholtz are verified.

* In Professor Joukowski's experiments, for $C_a = 0$ to 120, $C_{wo} = 8$ to 10, and in those of Mr. Betz, for a wing similar to No. 358 ($Vl = 1.4$), for $C_a = 0$ to 120, $C_{wo} = 3$ to 5, while wing 358, for $Vl = 2$ gives $C_{wo} = 1.5$ to 3. There is a discrepancy here, which is explained perhaps by the condition of the wing surface, but which should be investigated in order to determine whether it is not due to the method of experimenting, with wing elements placed between two vertical walls in a wind tunnel with continuous walls, thus producing the flow of infinite aspect ratio.

** See next page.

** Up to lift coefficients of 0.075 kgm²m : sec, wings 430 and 358 give results which seem incapable of improvement. All further researches should be devoted to devising ways to avoid deflections at high coefficients of lift. From the latter point of view, solutions similar to that of Handley-Page appear promising. Let us recall that the theory likewise renders it possible to trace the curve of the centers of lift and let us, in this connection, turn our attention to the shape of this curve for wing 431, which, instead of comprising, as for all the other wings, a neutral branch with positive coefficients of lift and a stable branch with negative coefficients of lift, consists, within the limits of the tests, of a neutral branch and a stable branch with positive coefficients of lift and a neutral branch with negative coefficients of lift.

Translated by National Advisory Committee for Aeronautics.

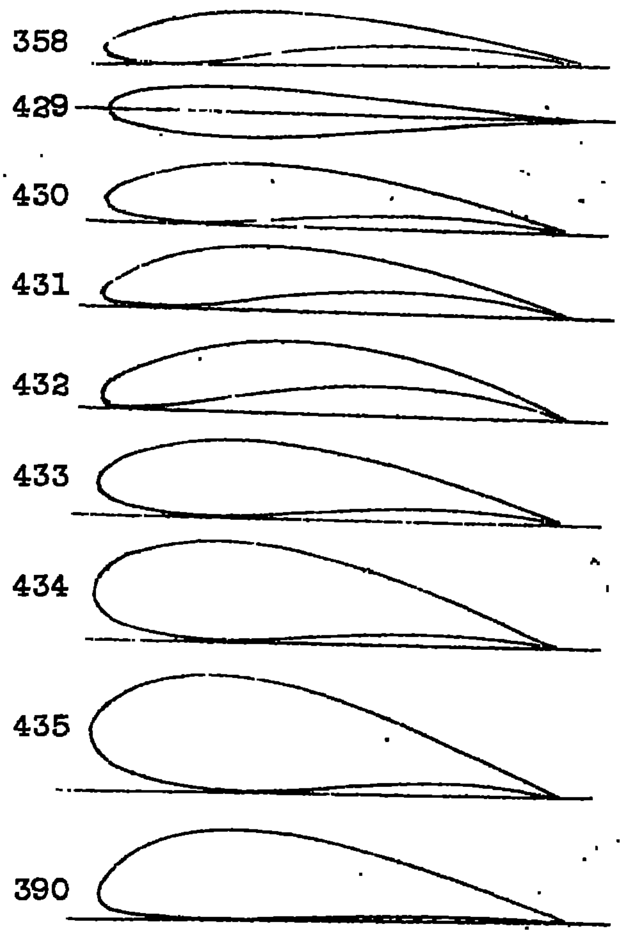

Fig. 1. Joukowski wing sections.

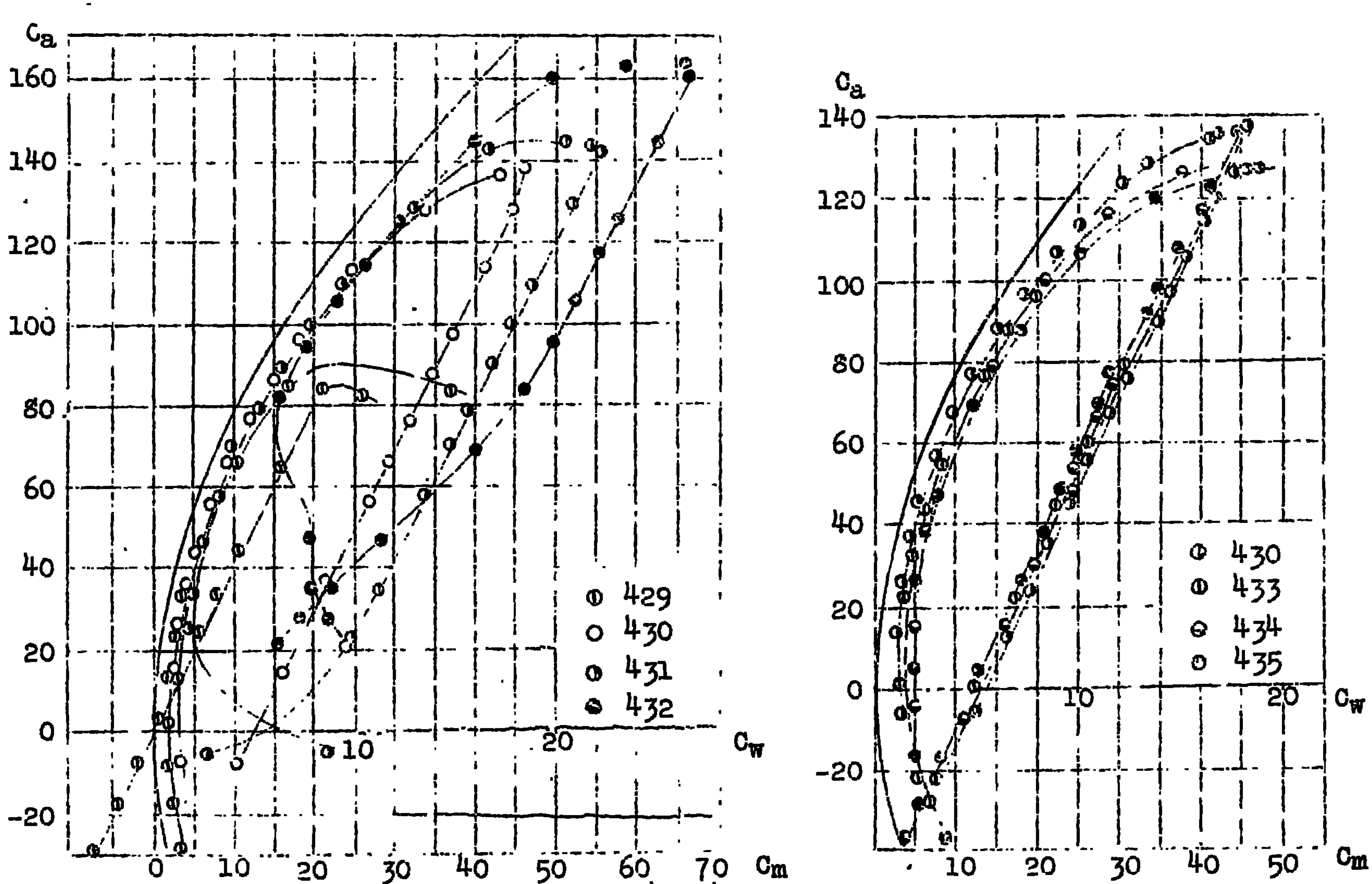

Fig.2. Polars of Joukowski wings Nos. 429-435. Aspect ratio of wings is 5.
Parabola of induced drag drawn on figure corresponds to this aspect ratio.

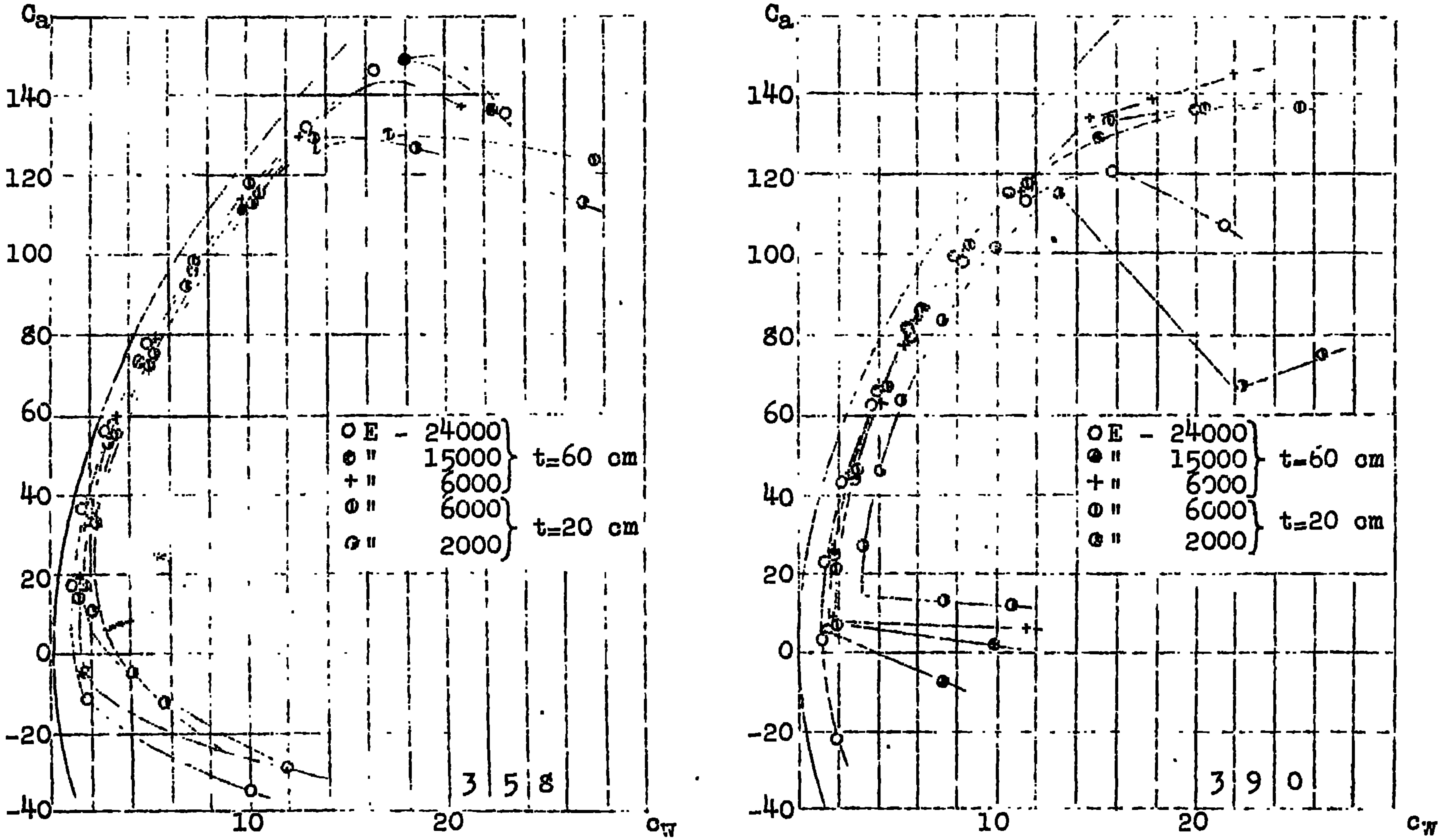

Fig. 3. Polars of wings Nos, 358 and 390.
E is the product of the chord in mm by the speed m.p.s. t is the wing chord which was 600 mm in the tests between vertical walls and 200 mm in the tests by the usual method.

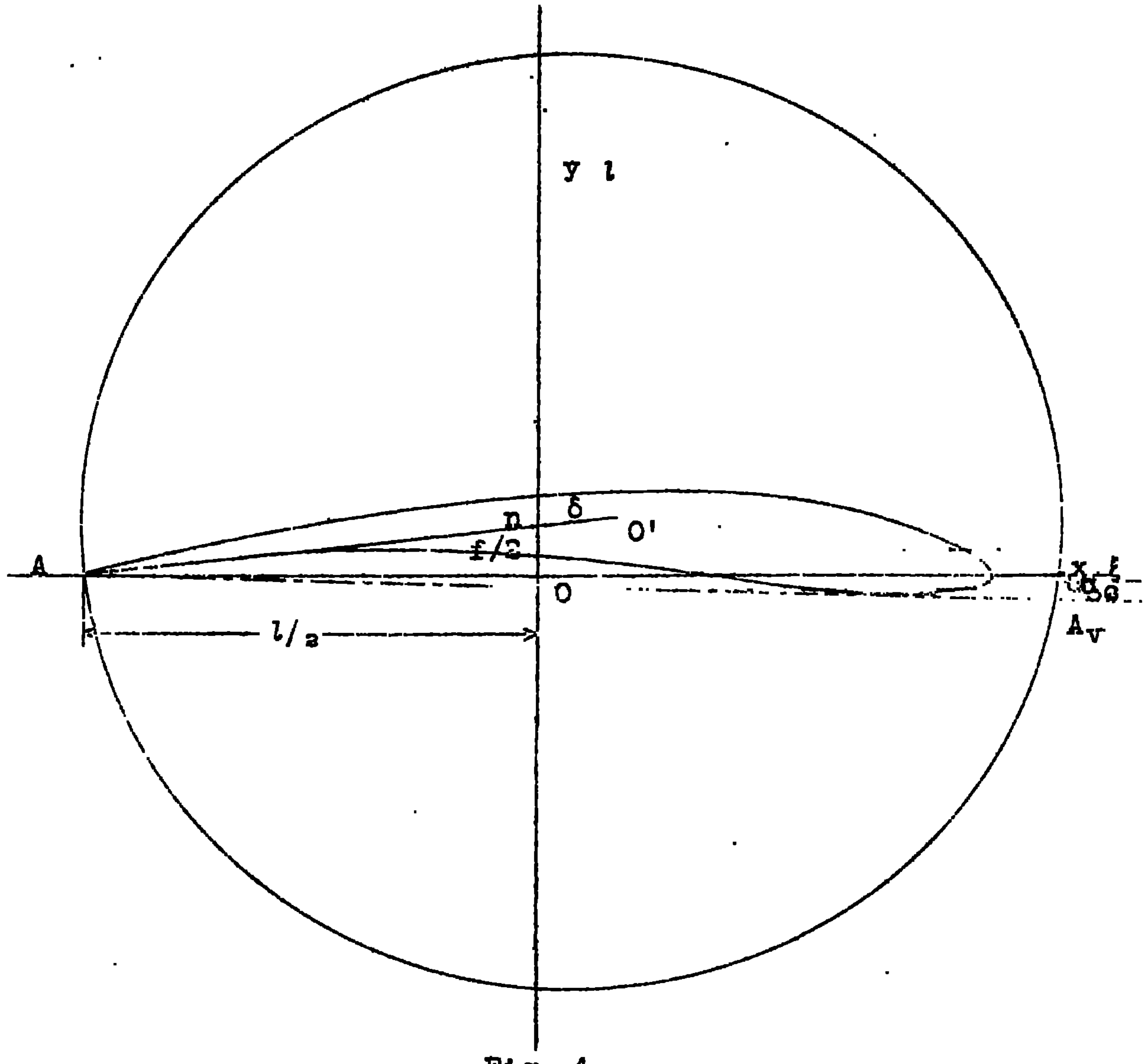

Fig. 4.

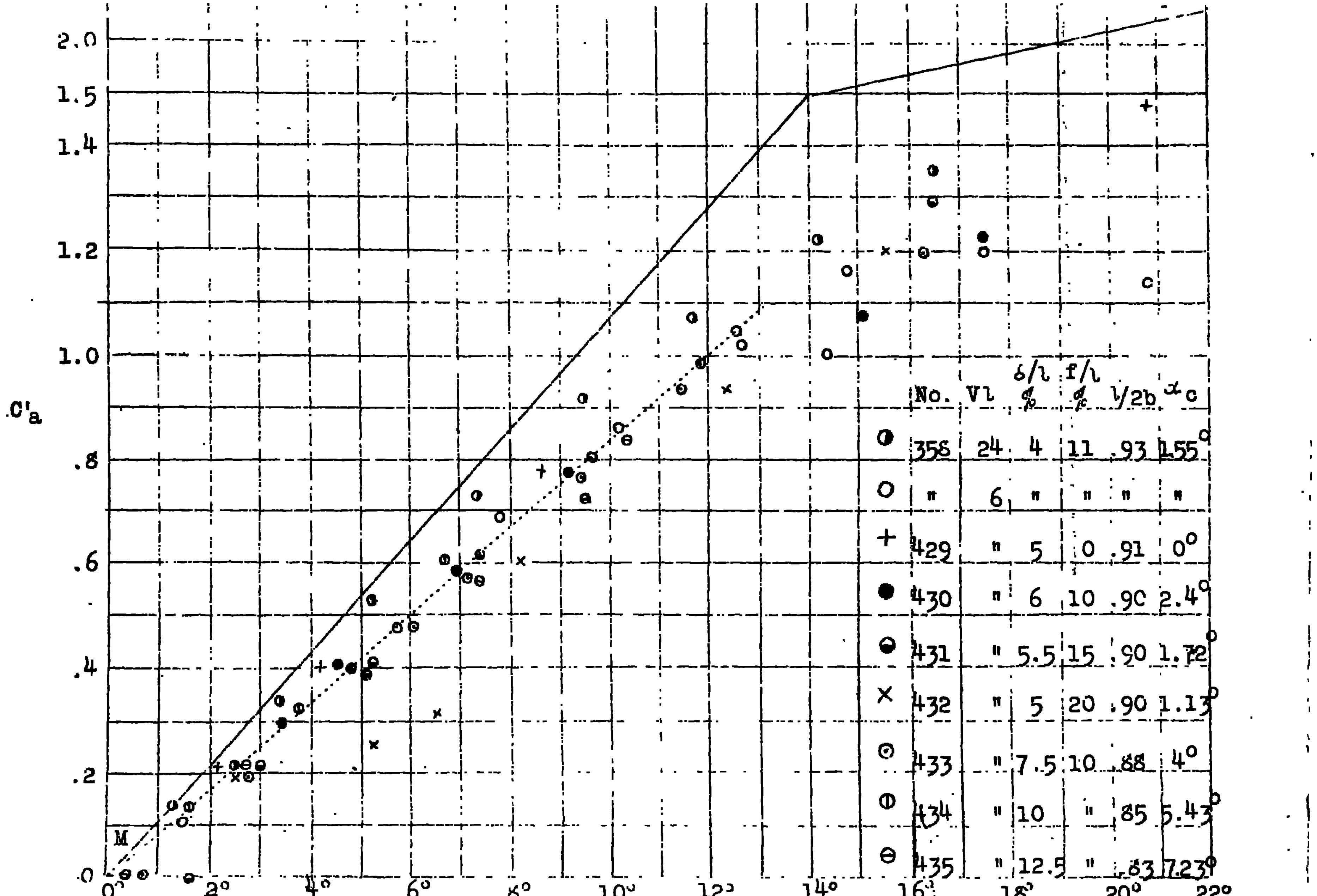

Fig.5 - Comparison of the experimental (arctan f/l + B) and theoretical coefficient of lift

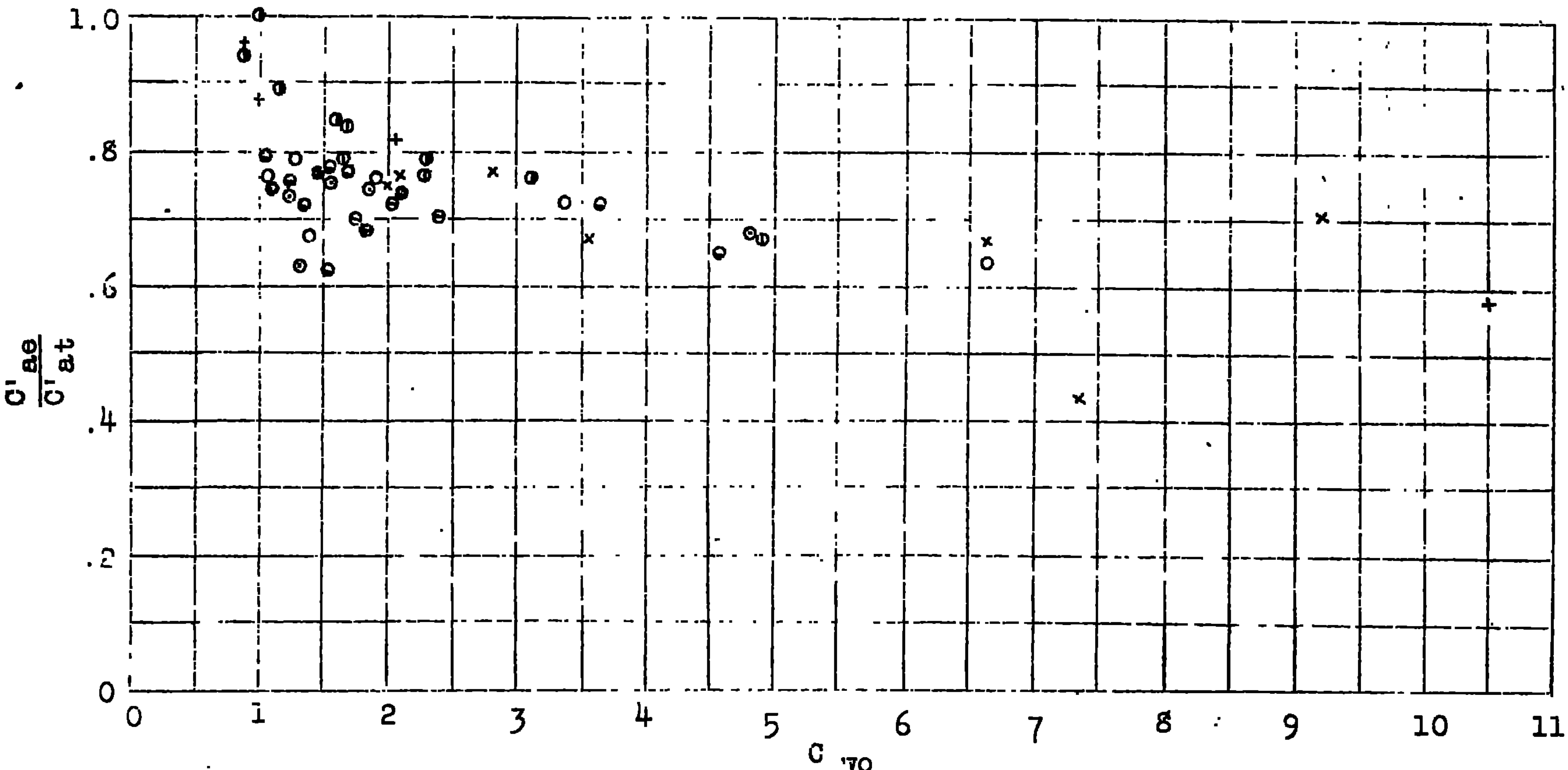

Fig.6 - Values of the ratio of the experimental (C'_{ae}) to the theoretical (C'_{at}) lift coefficient.

Note:- Wing designations are same as in Fig. 5.